Name________________________ Math 251-303

Final Examination

Part I: Short Answer

Instructions: *Place the answer in the space provided. Little or no partial credit will be given. You may use the back pages of your bluebook for scratch work; this work will NOT be examined.*

1. **(5 pts.)** What is the equation of the plane determined by the three points $A(1,0,2)$, $B(3,-1,6)$, $C(5,2,4)$?

Answer:

2. **(5 pts.)** What is the distance from the point $D(1,-2,3)$ to the plane determined by the points A, B, C in # 1?

Answer: *Distance* $=$

3. **(5 pts.)** Let C be the curve that is parametrically given by $\mathbf{R} = 3\sin t\,\mathbf{i} + 4t\,\mathbf{j} + 3\cos t\,\mathbf{k}$, $0 \le t \le \pi$. What is the vector $\mathbf{T}(t)$ tangent to $\mathbf{R}(t)$?

Answer: $\mathbf{T}(t) =$

4. **(5 pts.)** Consider the function $f(x,y,z) = e^{xy}\cos(x+z)$. What is the directional derivative for f at the point $P(0,-\pi/6,\pi/3)$ in the direction $\mathbf{u}$ parallel to $\mathbf{i} - \mathbf{j} + 2\,\mathbf{k}$?

Answer: $Df_{\mathbf{u}} =$

5. **(5 pts.)** What is $\frac{\partial w}{\partial u}$ if $w = e^x + xy^2$, $x = 3u - 2v$, and $y = 4e^{3u-2v}$? (Express your answer in terms of x and y.)

Answer: $\dfrac{\partial w}{\partial u} =$

6. **(8 pts.)** Set up integrals for the mass M and the moment $M_x = \iint y\delta\, dA$ for a thin plate that occupies the region inside the circle $r = 2\sin\theta$, but outside of the circle $r = 1$, if the density at any point P on the plate is given by $\delta(r,\theta) = r^{-3}$. Do NOT evaluate the integrals involved.

Answer:
$M =$
$M_x =$

7. **(5 pts.)** What is the integral formula in Stokes's Theorem?

Answer:

8. **(5 pts.)** What is $\nabla \times \mathbf{F}$, if $\mathbf{F} = y\,\mathbf{i} + xz\,\mathbf{j} + x^2\,\mathbf{k}$?

Answer: $\nabla \times \mathbf{F} =$

Part II: Essay Questions

Instructions: *Show all work in your bluebook.*

1. **(10 pts.)** A thin plate occupies the region $x^2/4 + y^2/9 \le 1$. If the temperature of the plate is $T(x, y) = x^2 + y^2 - 5y + 5$, find the coldest and hottest points on the plate. (Hint: use Lagrange multipliers to check the boundary.)

2. **(10 pts.)** A conical hole is drilled out of the top of a hemispherical piece of wood having radius $\rho = 2$. Given that the angle from the axis of the cone to its edge is $\pi/3$, and that the vertex of the cone is the center of the hemisphere, find the volume of the piece of wood that is left.

3. (10 pts.) Let C be the line segment from $P(1, 0, 2)$ to $Q(-2, 3, 1)$, and let $\mathbf{F}$ be the force given by $\mathbf{F}(x, y, z) = 2z\mathbf{i} - y\mathbf{j} + 2x\mathbf{k}$. Find the work done by $\mathbf{F}$ in moving a particle along C.

4. (12 pts.) Use Green's Theorem to find the counterclockwise circulation and outward flux for the vector field $\mathbf{F}(x, y) = xy\,\mathbf{i} + x^2\,\mathbf{j}$ and the curve C, where C is the boundary of the region enclosed by the parabola $y = x^2$ and $y = x$.

5. Let S be the surface of the solid hemisphere bounded by $x^2+y^2+z^2 = 4$ (where $z \geq 0$) and the plane, $z = 0$, and let $\mathbf{F} = 3x\,\mathbf{i} + 3y\,\mathbf{j} + 3z\,\mathbf{k}$.

(a) (9 pts.) Find Φ, the outward flux of $\mathbf{F}$ across S, by calculating the appropriate surface integral. (Hint: the surface integral has to be split into two pieces.)

(b) (6 pts.) Use the divergence theorem to calculate Φ.