A lowest order mixed finite element code for Forchheimer flow II

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1 Introduction

In the previous report, we presented the results of some numerical simulations on a single component flow model with a projection well. Our goal is to extend this model to the case of multi-component flow with multiple wells. Some of the wells will be injection wells, where fluid is reinjected into the porous medium. The injection process is expected to increase the pressure gradient in the medium and hence increase the production.

A model is currently being developed to simulate the situation where unwanted portion of the produced fluid is put back in to the medium at the injection well(s). For this, we need to keep track of the wanted component(s) of the fluid. This is done by a transport routine based on a Godunov-type method (see, e.g. [1], [2]).

In this note, we describe how the two codes, the mixed finite element routine discussed in the previous report and this transport routine, are combined to work together.

2 The model

Recall the Forchheimer flow equations

$$\frac{\partial(\phi\tilde{\rho})}{\partial t} + \nabla \cdot (u\tilde{\rho}) = q, \tag{1}$$

$$-(\nabla p + \rho g \nabla Z) = (\mu K^{-1} + \rho\beta|u|)u. \tag{2}$$

We assume the fluid has two components. Let $n_i, i = 1, 2$, be the mole fraction of the i-th component, i.e.

$$n_1 + n_2 = \tilde{\rho}. \tag{3}$$

In this test run, we set

$$n_1 = \alpha\tilde{\rho}, \quad n_2 = (1-\alpha)\tilde{\rho}, \tag{4}$$

where $0 < \alpha < 1$.

Now, by the conservation law, we get

$$\frac{\partial(\phi n_i)}{\partial t} + \nabla \cdot (u n_i) = q_i, \quad i = 1, 2, \tag{5}$$

where q_i is a source/sink term for the i-th component. Note that by summing over i, we get (1).

As was described in the previous report, q is calculated by the flow code (i.e. the mixed finite element code solving (1) and (2)) based on the well model. On the other hand, q_1, assuming that component 1 is the wanted material, is calculated by the transport code. q_2 is determined by $q - q_1$.

At each time step, the transport code uses $\phi(p)$, the porosity determined by the pressure, and u, the velocity of the fluid, calculated by the flow code. n_1 is initially given to the transport code as in (4). The transport code calculates the change of n_1 in time and produces q_1. In other words, n_1 is kept track of by the transport code independently once its initial state is given.

3 Test

In this test run, we assume the two components have the same physical properties, in particular the same velocity. This means that q_1/q should be equal to α and n_1 updated by the transport code should remain $\alpha\tilde{\rho}$ as time passes. α was set to 0.75 in this run.

The test was run in a square of side length 5000(ft) with no flow boundary condition. Other assumptions are as follows.

$$\begin{aligned} r_w &= 0.35(\text{ft}) \text{ (well radius)} \\ p_w &= 1000(\text{psi}) \text{ (bottom whole pressure)} \\ p_i &= 5000(\text{psi}) \text{ (initial pressure)} \end{aligned}$$

The code was run to t =2000(days). The number of timesteps of the flow code was 40 from $t = 0$ to $t = 400$ and 64 from $t = 400$ to $t = 2000$. The transport code takes smaller timesteps. It takes 35 to 40 timesteps in each of the 40 timesteps the flow code takes until $t = 400$ and 21 to 85 in each of the 64 timesteps thereafter. The number of timesteps the transport code takes is proportional to $|u|$ and as the velocity decreases, it takes less timesteps in a given time interval.

The plots of $\alpha\tilde{\rho}$ and n_1 on the diagonal at $t = 2000$ is attached. There is less than 1.5% difference between the two results and the source of this discrepancy is being investigated.

References

[1] S.K. Godunov, *A difference method for numerical calculation of discontinuous solutions of the equations of hydrodynamics*, Mat Sb., 47 (1959), pp.271-306.

[2] B. vanLeer, *Towards the ultimate conservative difference scheme II. Monotonicity and conservation combined in a second order scheme*, J. Comput. Phys., 14(1974), pp.263-275.

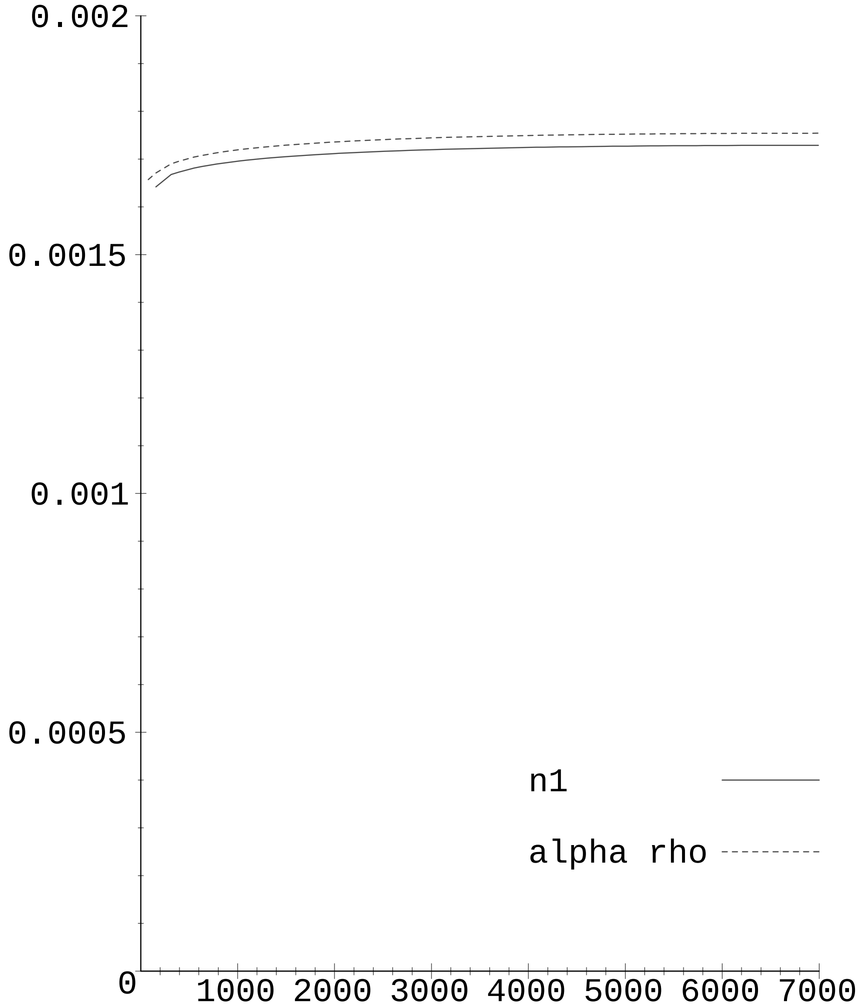

Figure 1: n_1 and $\alpha\tilde{\rho}$ on the diagonal after 2000 days